



In calculations all concentrations are expressed in μM , all rates in $\mu\text{M}\cdot\text{min}^{-1}$ and electrostatic and redox potentials in mV

I. INDEPENDENT MODEL VARIABLES

- | | |
|---------------------------------------|-------------------|
| a) protons concentration in matrix | $[\text{H}^+]_i$ |
| b) protons concentration in cytosol | $[\text{H}^+]_e$ |
| c) potassium concentration in matrix | $[\text{K}^+]_i$ |
| d) potassium concentration in cytosol | $[\text{K}^+]_e$ |
| e) phosphate concentration in matrix | $[\text{Pi}]_i$ |
| f) phosphate concentration in cytosol | $[\text{Pi}]_e$ |
| g) ADP concentration in cytosol | $[\text{ADP}]_e$ |
| h) ATP concentration in matrix | $[\text{ATP}]_i$ |
| i) ATP concentration in cytosol | $[\text{ATP}]_e$ |
| j) creatine phosphate concentration | $[\text{PCr}]$ |
| k) oxygen concentration | $[\text{O}_2]$ |
| l) NAD^+ concentration | $[\text{NAD}]_r$ |
| m) ubiquinol concentration | $[\text{UQ}]_r$ |
| n) reduced cytochrome c concentration | $[\text{cytc}]_r$ |
| o) transmembrane potential | $\Delta\Psi$ |

II. MODEL CONSTANTS

| | | |
|---|---------------|-------------------------|
| a) total adenine nucleotides concentration in matrix | $[AXP]_i$ | 16.26 mM |
| b) total adenine nucleotides concentration in cytosol | $[AXP]_e$ | 6.7 mM |
| c) creatine pool size | $[Cr_t]$ | 25 mM |
| d) total nicotinamide dinucleotide concentration | $[NAD_t]$ | 2.79 mM |
| e) total ubiquinone/ubiquinol concentration | $[UQ_t]$ | 1.35 mM |
| f) total cytochrome c concentration | $[cytc_t]$ | 0.27 mM |
| g) total cytochrome a concentration | $[cyta_t]$ | 0.135 mM |
| h) matrix magnesium concentration | $[Mg^{2+}]_i$ | 0.38 mM |
| i) cytosol magnesium concentration | $[Mg^{2+}]_e$ | 4 mM |
| j) phosphate dissociation constant | pK_{Pi} | 6.8 |
| k) ADP magnesium binding constant in matrix | $k_{d,ADP,i}$ | $282 \mu M^{-1}$ |
| l) ADP magnesium binding constant in cytosol | $k_{d,ADP,e}$ | $347 \mu M^{-1}$ |
| m) ATP magnesium binding constant in matrix | $k_{d,ATP,i}$ | $17 \mu M^{-1}$ |
| n) ATP magnesium binding constant in cytosol | $k_{d,ATP,e}$ | $24 \mu M^{-1}$ |
| o) NAD standard redox potential | $Em_{NAD,0}$ | -320 mV |
| p) UQ standard redox potential | $Em_{UQ,0}$ | 40 mV |
| q) cytochrome c standard redox potential | $Em_{cytc,0}$ | 250 mV |
| r) cytochrome a standard redox potential | $Em_{cyta,0}$ | 540 mV |
| s) ATP synthase proton stoichiometry | n_A | 2.5 |
| t) cell to mitochondrial volume ratio | R_{cm} | 4.35 |
| u) NAD buffering capacity coefficient | $buff_{NAD}$ | 5 |
| v) inner membrane electrical capacitance | C_m | $1 \mu M \cdot mV^{-1}$ |

III.DEPENDENT MODEL VARIABLES

- a) monovalent phosphate ion concentration in matrix $[Pi]_{i,j} = \frac{[Pi]_i}{1 + 10^{(pH_i - pK_{pi})}}$
- b) monovalent phosphate ion concentration in cytosol $[Pi]_{e,j} = \frac{[Pi]_e}{1 + 10^{(pH_e - pK_{pi})}}$
- c) AMP concentration in cytosol $[AMP]_e = [AXP]_e - ([ATP]_e + [ADP]_e)$
- d) ADP concentration in matrix $[ADP]_i = [AXP]_i - [ATP]_i$
- e) free ADP in matrix $[ADP]_{i,f} = \frac{[ADP]_i}{1 + \frac{[Mg^{2+}]_i}{k_{d,ADP,i}}}$
- f) magnesium bound ADP in matrix $[ADP]_{i,m} = [ADP]_i - [ADP]_{i,f}$
- g) free ADP in cytosol $[ADP]_{e,f} = \frac{[ADP]_e}{1 + \frac{[Mg^{2+}]_e}{k_{d,ADP,e}}}$
- h) magnesium bound ADP in cytosol $[ADP]_{e,m} = [ADP]_e - [ADP]_{e,f}$
- i) free ATP in matrix $[ATP]_{i,f} = \frac{[ATP]_i}{1 + \frac{[Mg^{2+}]_i}{k_{d,ATP,i}}}$
- j) magnesium bound ATP in matrix $[ATP]_{i,m} = [ATP]_i - [ATP]_{i,f}$
- k) free ATP in cytosol $[ATP]_{e,f} = \frac{[ATP]_e}{1 + \frac{[Mg^{2+}]_e}{k_{d,ATP,e}}}$
- l) magnesium bound ATP in cytosol $[ATP]_{e,m} = [ATP]_e - [ATP]_{e,f}$
- m) creatine concentration $[Cr] = [Cr_t] - [PCr]$
- n) NAD redox potential $Em_{NAD} = Em_{NAD,0} + \frac{1}{2} \cdot \frac{R \cdot T}{F} \cdot \ln \left(\frac{[NAD_o]}{[NAD_r]} \right)$
- o) NADH concentration $[NAD_o] = [NAD_t] - [NAD_r]$
- p) ubiquinone redox potential $Em_{UQ} = Em_{UQ,0} + \frac{1}{2} \cdot \frac{R \cdot T}{F} \cdot \ln \left(\frac{[UQ_o]}{[UQ_r]} \right)$
- q) ubiquinone concentration $[UQ_o] = [UQ_t] - [UQ_r]$
- r) cytochrome c redox potential $Em_{cytc} = Em_{cytc,0} + \frac{R \cdot T}{F} \cdot \ln \left(\frac{[cytc_o]}{[cytc_r]} \right)$
- s) oxidized cytochrome c concentration $[cytc_o] = [cytc_t] - [cytc_r]$

III. DEPENDENT MODEL VARIABLES

t) cytochrome a redox potential

$$Em_{\text{cyta}} = Em_{\text{cytc}} + \Delta p - \Delta \Psi$$

u) reduced cytochrome a concentration

$$[\text{cyta}_r] = \frac{[\text{cyta}_t]}{1 + e^{\frac{F}{R \cdot T} (Em_{\text{cyta}} - Em_{\text{cyta},0})}}$$

v) oxidized cytochrome a concentration

$$[\text{cyta}_o] = [\text{cyta}_t] - [\text{cyta}_r]$$

w) H⁺ buffering capacity coefficient for matrix

$$\text{buff}_{\text{H}_i^+} = 0.022 \cdot \left(\frac{10^{-\text{pH}_i} - 10^{-\text{pH}_i - 0.001}}{0.001} \right)^{-1}$$

x) H⁺ buffering capacity coefficient for cytosol

$$\text{buff}_{\text{H}_e^+} = 0.025 \cdot \left(\frac{10^{-\text{pH}_e} - 10^{-\text{pH}_e - 0.001}}{0.001} \right)^{-1}$$

IV. EXTRAMITOCHONDRIAL PROCESSES

a) ATP consumption

$$v_{\text{cons}} = \frac{k_{\text{cons}}}{1 + \frac{K_{\text{m}_{\text{cons,ATP}}}}{[\text{ATP}]_e}}$$

$$k_{\text{cons}} = 13937 \mu\text{M} \cdot \text{min}^{-1}$$

$$K_{\text{m}_{\text{cons,ATP}}} = 150 \mu\text{M}$$

b) creatine kinase

$$v_{\text{CK}} = k_{\text{CK},f} \cdot [\text{ADP}]_e \cdot [\text{PCr}] - k_{\text{CK},b} \cdot [\text{ATP}]_e \cdot [\text{Cr}]$$

$$k_{\text{CK},f} = 6.60560 \mu\text{M}^{-1} \cdot \text{min}^{-1}$$

$$k_{\text{CK},b} = 0.0030025441 \mu\text{M}^{-1} \cdot \text{min}^{-1}$$

c) adenylate kinase

$$v_{\text{AK}} = k_{\text{AK},f} \cdot [\text{ADP}]_e \cdot [\text{ADP}]_{e,m} - k_{\text{AK},b} \cdot [\text{ATP}]_{e,m} \cdot [\text{AMP}]_e$$

$$k_{\text{AK},f} = 2956.98 \mu\text{M}^{-1} \cdot \text{min}^{-1}$$

$$k_{\text{AK},b} = 78.0208 \mu\text{M}^{-1} \cdot \text{min}^{-1}$$

d) glycolysis

$$v_{\text{glyc}} = 0.2 \cdot v_{\text{DH}}$$

e) proton efflux

$$v_{\text{EFF}} = k_{\text{EFF}} \cdot (7.0 - \text{pH}_e)$$

$$k_{\text{EFF}} = 10000 \mu\text{M} \cdot \text{min}^{-1} \cdot \text{pH unit}^{-1}$$

V. INTRAMITOCHONDRIAL PROCESSES

a) substrate dehydrogenation

$$v_{\text{DH}} = k_{\text{DH}} \cdot \frac{1}{1 + \frac{K_{\text{m}_{\text{DH,NAD}}}}{\left(\frac{[\text{NAD}_o]}{[\text{NAD}_r]} \right)^{\text{pD}_{\text{DH}}}}}$$

$$k_{\text{DH}} = 96293 \mu\text{M} \cdot \text{min}^{-1}$$

$$K_{\text{m}_{\text{DH,NAD}}} = 100$$

$$\text{pD}_{\text{DH}} = 0.8$$

VI. INNER MITOCHONDRIAL MEMBRANE PROCESSES

- a) complex I
- $$v_{C1} = k_{C1} \cdot \Delta G_{C1}$$
- $$k_{C1} = 819.61 \mu\text{M} \cdot \text{min}^{-1} \cdot \text{mV}^{-1}$$
- $$\Delta G_{C1} = E_{m_{UQ}} - E_{m_{NAD}} - 2 \cdot \Delta p$$
- b) complex III
- $$v_{C3} = k_{C3} \cdot \Delta G_{C3}$$
- $$k_{C3} = 467.9 \mu\text{M} \cdot \text{min}^{-1} \cdot \text{mV}^{-1}$$
- $$\Delta G_{C3} = E_{m_{cytc}} - E_{m_{UQ}} - 2 \cdot \Delta p + \Delta \Psi$$
- c) complex IV
- $$v_{C4} = k_{C4} \cdot [\text{cytc}_r] \cdot [\text{cyta}_r] \cdot \frac{1}{1 + \frac{K_{m_{C4, O_2}}}{[O_2]}}$$
- $$k_{C4} = 24.696 \mu\text{M}^{-1} \cdot \text{min}^{-1}$$
- $$K_{m_{C4, O_2}} = 150 \mu\text{M}$$
- d) proton leak
- $$v_{LK} = k_{LK,1} \cdot (e^{k_{LK,2} \cdot \Delta p} - 1)$$
- $$k_{LK,1} = 8.5758 \mu\text{M} \cdot \text{min}^{-1}$$
- $$k_{LK,2} = 0.038 \text{mV}^{-1}$$
- e) ATP synthase
- $$v_{SN} = k_{SN} \cdot \frac{\gamma_{SN} - 1}{\gamma_{SN} + 1}$$
- $$k_{SN} = 117706 \mu\text{M} \cdot \text{min}^{-1}$$
- $$\ln(\gamma_{SN}) = n_A \cdot \frac{F \cdot \Delta p}{R \cdot T} - \left(\frac{\Delta G_{SN,0}}{R \cdot T} + \ln \left(\frac{[\text{ATP}]_i \cdot 10^6 \mu\text{M}}{[\text{ADP}]_i \cdot [\text{Pi}]_i} \right) \right)$$
- $$\Delta G_{SN,0} = 31.9 \text{kJ} \cdot \text{mol}^{-1}$$
- f) adenine nucleotides antiport
- $$v_{ANT} = k_{ANT} \cdot \frac{1}{1 + \frac{K_{m_{ANT, ADP}}}{[\text{ADP}]_{e,f}}}$$
- $$\cdot \left(\frac{[\text{ADP}]_{e,f}}{[\text{ADP}]_{e,f} + [\text{ATP}]_{e,f} \cdot e^{-\frac{F}{R \cdot T} \cdot (1-f_{ANT}) \cdot \Delta \Psi}} - \frac{[\text{ADP}]_{i,f}}{[\text{ADP}]_{i,f} + [\text{ATP}]_{i,f} \cdot e^{-\frac{F}{R \cdot T} \cdot f_{ANT} \cdot \Delta \Psi}} \right)$$
- $$k_{ANT} = 187185 \mu\text{M} \cdot \text{min}^{-1}$$
- $$K_{m_{ANT, ADP}} = 3.5 \mu\text{M}$$
- $$f_{ANT} = 0.65$$
- g) phosphate translocator
- $$v_{PiT} = k_{PiT} \cdot ([\text{Pi}]_{e,j} \cdot [\text{H}^+]_e - [\text{Pi}]_{i,j} \cdot [\text{H}^+]_i)$$
- $$k_{PiT} = 238.1 \mu\text{M}^{-1} \cdot \text{min}^{-1}$$
- h) potassium uniport
- $$v_{Kuni} = k_{Kuni} \cdot \left([\text{K}^+]_e \cdot e^{\frac{F}{R \cdot T} \cdot \frac{-\Delta \Psi}{2}} - [\text{K}^+]_i \cdot e^{\frac{F}{R \cdot T} \cdot \frac{\Delta \Psi}{2}} \right)$$
- $$k_{Kuni} = 0.00001 \text{min}^{-1}$$
- i) H⁺/K⁺ exchanger
- $$v_{KHex} = k_{KHex} \cdot ([\text{K}^+]_i \cdot [\text{H}^+]_e - [\text{K}^+]_e \cdot [\text{H}^+]_i)$$
- $$k_{KHex} = 0.003452 \mu\text{M}^{-1} \cdot \text{min}^{-1}$$

VII. DIFFERENTIAL EQUATIONS

- a) protons concentration in matrix $[H^+]_i = \left(\begin{array}{c} -4 \cdot v_{C1} - 2 \cdot v_{C3} - 4 \cdot v_{C4} + \\ + n_A \cdot v_{SN} + v_{PiT} + v_{LK} + v_{KHex} \end{array} \right) \cdot R_{cm} \cdot \text{buff}_{H^+}^{-1}$
- b) protons concentration in cytosol $[H^+]_e = \left(\begin{array}{c} 4 \cdot v_{C1} + 4 \cdot v_{C3} + 2 \cdot v_{C4} + \\ - n_A \cdot v_{SN} - v_{PiT} - v_{LK} - v_{KHex} - v_{EFF} \end{array} \right) \cdot \text{buff}_{H^+}^{-1}$
- c) potassium concentration in matrix $[K^+]_i = (v_{Kuni} - v_{KHex}) \cdot R_{cm}$
- d) potassium concentration in cytosol $[K^+]_e = 0$
- e) phosphate concentration in matrix $[Pi]_i = (v_{PiT} - v_{SN}) \cdot R_{cm}$
- f) phosphate concentration in cytosol $[Pi]_e = v_{cons} - v_{PiT} - v_{glyc}$
- g) ADP concentration in cytosol $[ADP]_e = v_{cons} - v_{ANT} - 2 \cdot v_{AK} - v_{CK} - v_{glyc}$
- h) ATP concentration in matrix $[ATP]_i = (v_{SN} - v_{ANT}) \cdot R_{cm}$
- i) ATP concentration in cytosol $[ATP]_e = v_{ANT} - v_{cons} + v_{AK} + v_{CK} + v_{glyc}$
- j) creatine phosphate concentration $[PCr] = -v_{CK}$
- k) oxygen concentration $[O_2] = 0$
- l) NAD⁺ concentration $[NAD_r] = (v_{DH} - v_{C1}) \cdot R_{cm} \cdot \text{buff}_{NAD}^{-1}$
- m) ubiquinol concentration $[UQ_r] = (v_{C1} - v_{C3}) \cdot R_{cm}$
- n) reduced cytochrome c concentration $[cytc_r] = (v_{C3} - v_{C4}) \cdot R_{cm}$
- o) transmembrane potential $\Delta\psi = \left(\begin{array}{c} -4 \cdot v_{C1} - 2 \cdot v_{C3} - 4 \cdot v_{C4} + \\ + n_A \cdot v_{SN} + v_{ANT} + v_{LK} + v_{Kuni} \end{array} \right) \cdot R_{cm} \cdot C_m^{-1}$